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Steady and transient thermal convection in a fluid layer with uniform volumetric energy sources

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Measurements of the overall heat flux in steady convection have been made in a horizontal layer of dilute aqueous electrolyte. The layer is bounded below by a rigid zero-heat-flux surface and above by a rigid isothermal surface. Joule heating by an alternating current passing horizontally through the layer provides a uniformly distributed volumetric energy source. The Nusselt number at the upper surface is found to be proportional to $Ra^{0\cdot227}$ in the range $1\cdot4 \leqslant Ra/Ra_c \leqslant 1\cdot6 \times 10^9$, which covers the laminar, transitional and turbulent flow regimes. Eight discrete transitions in the heat flux are found in this Rayleigh number range. Extrapolation of the heat-transfer correlation to the conduction value of the Nusselt number yields a critical Rayleigh number which is within $-6\cdot7$ % of the value given by linearized stability theory. Measurements have been made of the time scales of developing convection after a sudden start of volumetric heating and of decaying convection when volumetric heating is suddenly stopped. In both cases, the steady-state temperature difference across the layer appears to be the controlling physical parameter, with both processes exhibiting the same time scale for a given steady-state temperature difference, or $|\Delta Ra|$. For step changes in Ra such that $|\Delta Ra| > 100Ra_c$, the time scales for both processes can be represented by $Fo \propto |\Delta Ra|^m$, where Fo is the Fourier number of the layer. Temperature profiles of developing convection exhibit a temperature excess in the upper 15–20 % of the layer in the early stages of flow development for Rayleigh numbers corresponding to turbulent convection. This excess disappears when the average core temperature becomes large enough to permit eddy transport and mixing processes near the upper surface. The steady-state limits in the transient experiments yield heat-transfer data in agreement with the results of the steady-state experiments.

1. Introduction

This paper presents the results of an experimental investigation of thermal convection in a horizontal layer of fluid containing uniformly distributed volumetric energy sources. The layer is bounded below by a rigid zero-heat-flux surface and above by a rigid isothermal surface. Layers which are thin relative to their horizontal extent are considered to keep edge effects and the effects of secondary flows to a minimum. In the first part of the study, a correlation between the steady-state Nusselt number at the upper surface and the Rayleigh number is determined for a range of Rayleigh numbers extending from creeping flow ($Ra \gtrsim Ra_c$, where Ra_c is the critical Rayleigh number for the onset of convection given by linearized stability theory) to highly turbulent convection. The second part of the study concerns the nature of developing (also decaying) convection when the volumetric energy generation rate undergoes a finite step change. The objective of this work is to determine the time needed for the layer to reach a steady state, i.e. either convection or no motion, as a function of the step change in Rayleigh number.

The motivation for the present work arises, in part, from the belief that thermal convection driven by heat sources plays an important role in the convective processes in the earth's mantle (Knopoff 1967; McKenzie, Roberts & Weiss 1974; Richter 1973; Runcorn 1962; Turcotte & Oxburgh 1972) and is an important aspect of the post-accident heat removal problem (Carbenier *et al* 1975) that can result in the event of a core meltdown in a nuclear power reactor. In both of these areas, heat-transfer data are needed for verification of analytical and numerical predictions of the steady-state and transient convective processes and for making computations of maximum temperature differences and upward heat fluxes. Experiments in a horizontal layer provide such data, albeit in a system which is an idealization of the complex geophysical and technological problems. Since there has not apparently been an experimental study of the heat-transfer problem for Rayleigh numbers which span the laminar and highly turbulent regimes of convection, additional motivation for the present work arises from a desire to provide such information.

The studies of Roberts (1967) and Thirlby (1970) appear to be the only existing theoretical work on the laminar heat-transport problem in a layer of fluid of moderate Prandtl number. Roberts was concerned primarily with the stability of the planform of the convective motion. In developing his analysis, he determined the heat flux at the upper surface using the 'shape-factor' approximation (Stuart 1958) and the 'mean-field' approximation (Roberts 1966). The heat-flux predictions obtained via the shape-factor approximation were independent of both the Prandtl number and the planform, whereas for flows deemed stable via the mean-field approach, a complicated interdependence was found among the planform of perturbations applied to the basic flow, the Rayleigh number and the Prandtl number. Thirlby's study was a numerical analysis of the laminar problem with major emphasis placed on the determination of the planform and other flow characteristics as a function of the Rayleigh and Prandtl numbers. He computed the steady-state temperature and velocity fields as the limit of the unsteady problem using the method of artificial compressibility. Heat-transport results were computed for convection in rolls at the critical wavenumber and compared with Roberts' results. Good agreement was obtained with Roberts' heat fluxes for $Ra \lesssim 3Ra_c$. In addition, Thirlby's three-dimensional solutions at the critical wavenumber were found to be in good agreement with those for roll convection.

McKenzie *et al.* (1974) also considered the laminar problem, but for fluids of very high Prandtl number such as are encountered in geophysical convection. Their numerical experiments were concerned primarily with predictions of the stream-function (i.e. velocity) and temperature fields for an infinite Prandtl number fluid and Rayleigh numbers up to approximately 10^6. A square region with stress-free horizontal and vertical boundaries was used in the computations. Their paper did not present heat-transfer results in the form of the Nusselt number as a function of the Rayleigh number. A boundary-layer analysis of the governing equations based on the physical features of the computed flow and temperature fields showed that $Nu \propto Ra^{\frac{1}{5}}$ for high Rayleigh number convection.

Fiedler & Wille (1971) analytically investigated the turbulent convection problem.

This study	Kulacki & Nagle (1975)
$4 \cdot 2 \times 10^5$	$4 \cdot 2 \times 10^5$
$6 \cdot 1 \times 10^6$	$4 \cdot 9 \times 10^6$
$2 \cdot 0 \times 10^7$	$3 \cdot 0 \times 10^7$
$3 \cdot 3 \times 10^8$	$4 \cdot 0 \times 10^8$
$1 \cdot 5 \times 10^9$	$1 \cdot 4 \times 10^9$
$1 \cdot 5 \times 10^{10}$	
$6 \cdot 5 \times 10^{10}$	
$3 \cdot 6 \times 10^{11}$	

TABLE 1. Points of discrete transition in heat flux.

They made use of the Prandtl mixing-length hypothesis to close the turbulent energy equation. Their prediction of the overall Nusselt number thus relied on empirical constants introduced in the analysis for which definitive measurements were lacking.

Early experimental work on thermal convection in horizontal layers with the thermal and hydrodynamic boundary conditions of interest in the present study was largely qualitative in nature. The investigations of Tritton & Zarraga (1967) and Schwiderski & Schwab (1971) were flow-visualization studies of the planform of convection in the immediate post-stability regime ($Ra \gtrsim Ra_c$) and for Rayleigh numbers up to the transition to turbulence. Both studies employed thin electrolytically heated water layers and suspended particles for flow visualization. Cellular planforms were observed in both studies with downflow in the centres of the cells (counter to that of Bénard convection in liquids) for $Ra \lesssim 80Ra_c$. The planform was found to be quasi-steady in this Rayleigh number range. For $Ra > 80Ra_c$, local turbulent motions were observed, and cell breakup became abrupt and random over the layer.

Experimental work on the heat-transport problem of interest to the present study is limited to the investigations of Fiedler & Wille (1971) and Kulacki & Nagle (1975). Both studies employed electrolytically heated water layers. Fiedler & Wille measured the average Nusselt number at the upper surface as a function of the Rayleigh number. Their correlation for heat transfer was given by

$$\left.\begin{aligned} Nu &= 0 \cdot 526\, Ra^{0 \cdot 228} \\ \text{for} \quad 2 \times 10^5 &\leqslant Ra \leqslant 6 \times 10^8, \quad 0 \cdot 29 \leqslant L/X \leqslant 1 \cdot 65. \end{aligned}\right\} \tag{1}$$

No Prandtl number range was reported by Fiedler & Wille but $6 \lesssim Pr \lesssim 7$ could be assumed for their water–salt solution. Kulacki & Nagle (1975) measured overall Nusselt numbers for a range of Rayleigh numbers which overlapped that of the Fiedler & Wille study. Their correlation for the heat-transfer coefficient at the upper surface was

$$\left.\begin{aligned} Nu &= 0 \cdot 305\, Ra^{0 \cdot 239} \\ \text{for} \quad 1 \cdot 5 \times 10^5 &\leqslant Ra \leqslant 2 \cdot 5 \times 10^9, \\ 6 \cdot 21 \leqslant Pr &\leqslant 6 \cdot 64, \quad 0 \cdot 05 \leqslant L/X \leqslant 0 \cdot 25. \end{aligned}\right\} \tag{2}$$

(A definition of the dimensionless groups in (1) and (2) is given in § 3.)

The data of Kulacki & Nagle also permitted determination of five discrete transitions in the heat flux (see table 1). Limited measurements of fluctuating temperatures in turbulent convection as a function of vertical position in the layer were in qualitative agreement with interferometric observations by Kulacki & Goldstein (1972) in a layer with two constant-temperature boundaries. Temperature fluctuations were found to be of large amplitude and high intensity just outside the thermal boundary layer on the upper surface. Kulacki & Nagle also conducted an experiment on the nature of the transient response of the layer when it was subjected to a step input of power. With an initially motionless isothermal layer, they found that, when internal heating was suddenly started such that $Ra = 9{\cdot}3 \times 10^7$, the temperature profile exhibited a local excess, or overshoot, near the upper surface. The remainder of the layer, however, immediately developed a nearly constant temperature, which is characteristic of turbulent convection. The temperature excess did not exist when steady-state convection was reached. It is desirable to have additional experiments to confirm the temperature overshoot feature of transient convection observed by Kulacki & Nagle. This provides further motivation for the transient experiments conducted in the present study.

2. Apparatus and procedure

Since steady and transient heat-transfer data were desired over a wide range of Rayleigh numbers, two convection chambers were used. For the high Rayleigh number experiments, the apparatus of Kulacki & Nagle (1975) was employed. Low Rayleigh number experiments were conducted with a smaller chamber to permit more accurate measurements of temperature differences and better overall thermal control of the convecting layer. Only a brief description of the experimental apparatus will be given here. Further details of the design and construction of the apparatus are given by Kulacki & Nagle (1975) and Kulacki & Emara (1976).

Both convection chambers comprised a horizontal layer of dilute aqueous silver nitrate solution† bounded above by a plate of constant temperature and below by an insulated surface. The top plate contained channels in a double spiral pattern for circulating cooling water which was maintained at a constant temperature to within $\pm 0{\cdot}05$ °C. A sheet of Mylar covered the top plate to insulate it electrically from the fluid layer. The layer dimensions were $25{\cdot}4 \times 25{\cdot}4$ cm for the small apparatus and $50{\cdot}8 \times 50{\cdot}8$ cm for the large apparatus. Two of the side walls of each convection chamber contained silver-plated electrodes for passing a 60 Hz alternating electric current through the fluid. The other side walls and the lower surface were Plexiglas. A guard heater was imbedded in the lower surface to maintain zero heat flux at the lower boundary of the fluid layer. Temperature differences across the layer were measured with thermocouples in the upper and in the lower boundaries. Several

† The percentage weight of silver nitrate in all experiments was 0·5 % (0·02 molar) or less. This electrolyte concentration is much lower than that in the experiments of Tritton & Zarraga (1967) and Schwiderski & Schwab (1971) and mitigates the effects of non-uniformities in the internal energy generation due to temperature effects. For convection in the laminar regime, where the flow field is not truly steady (Kulacki & Goldstein 1972), the effects of non-uniformities in internal energy generation are thus considered negligible because overall temperature differences across the layer are small (0·5 °C or less). In turbulent convection, eddy mixing acts to reduce non-uniformities in internal energy generation. Furthermore, the low electrolyte concentration tends to eliminate the current *vs.* time behaviour observed by Schwiderski & Schwab (1971) as an important factor in the present steady and transient experiments.

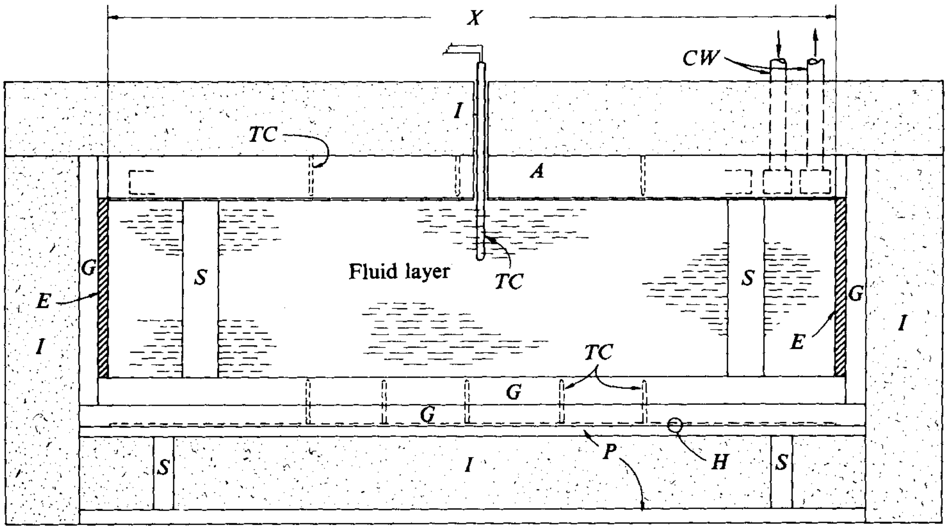


FIGURE 1. Cross-sectional view of convection chamber. $X = 25{\cdot}4$ cm for the small apparatus and 50·8 cm for the large apparatus. A, aluminium; CW, cooling water; E, electrode; G, Plexiglas; H, resistance heater; I, insulation; P, plywood; TC, thermocouple; S, spacer.

thermocouples were imbedded in each of the surfaces so that a spatially averaged temperature could be measured. A diagram of the convection chamber is presented in figure 1.

For measurement of temperature within the fluid in the transient experiments, a small glass-encased thermocouple probe was inserted into the layer through the top plate. The thermocouple junction was 0·01 cm in diameter, and the glass tube containing the thermocouple leads was nominally 0·19 cm in diameter. The response of the thermocouple was sufficiently rapid (< 1 s for a 1 °C step input) for measurement of the temperature transients encountered. Mean temperatures of steady convection were obtained from a graphical average of a strip-chart record of the probe output over a 1 h period.

Power to the layer was supplied from a line voltage regulator and transformer. For the low Rayleigh number experiments, power consumption was measured with a wattmeter transducer employing a Hall element. A voltmeter and ammeter were used for power measurements in the high Rayleigh number runs. The maximum uncertainty in power consumption in both convection chambers was 1 %, and the power was varied from 0·34 to 2468 W for the range of Rayleigh numbers of the study.

Prior to each run, the fluid layer was brought to a constant temperature slightly below room temperature. This procedure was followed so that the mean temperature in the convecting layer would be approximately equal to the ambient temperature, thus keeping heat losses to a minimum. (This was especially important for the low Rayleigh number experiments.) Power was then applied to the layer, and the flow was allowed to develop for a time much greater than that required for development of the conduction temperature profile at the given level of power input. For the steady-state experiments, flow development times of from 3 to 12 h were allowed, depending on the layer depth. After a steady state had been reached, several readings of the temperatures at the upper and lower surfaces were made. Another set of temperature readings was taken half an hour later, and when successive sets of readings were in agreement the data were recorded.

A similar procedure was followed in the transient experiments except that the attainment of the steady state was determined from a strip-chart record of the probe response. Temperature data for developing and decaying convection were obtained by recording the response of the probe at various vertical positions in the layer. The temperature histories obtained thus permitted construction of temperature profiles as a function of time at a given value of the Rayleigh number.

3. Results

Steady-state experiments

The maximum temperature differences in steady convection, $\Delta T = T_0 - T_1$, where T_0 is the temperature of the lower surface and T_1 is the temperature of the upper surface, are presented in figure 2 as a function of the Rayleigh number. In this figure, the quantity ΔT is normalized by the maximum temperature difference for conduction, $HL^2/2k$, where H is the volumetric rate of energy generation, L is the layer depth and k is the thermal conductivity. Included in figure 2 are the data of Kulacki & Nagle (1975) and the steady-state temperature differences in the transient experiments. The temperature differences for the various experimental studies are in good agreement over the entire range of Rayleigh numbers. For Rayleigh numbers corresponding to turbulent convection, i.e. for $Ra > 100Ra_c$, where $Ra_c = 1344$ (see Kulacki & Goldstein 1975; and below), the data show that convective mixing effectively eliminates almost all of the temperature difference that would exist for purely conductive energy transport.

The heat-transfer data were correlated in terms of the Nusselt number at the upper surface and the Rayleigh number:

$$Nu = hL/k, \quad Ra = (g\beta L^3/\alpha\nu)(HL^2/2k),$$

where h is the film coefficient for heat transfer based on the power input to the layer and ΔT, g is the gravitational acceleration, α is the thermal diffusivity and ν is the kinematic viscosity.

A correlation of the form

$$Nu = \text{constant} \times Ra^m$$

was assumed, and the thermophysical properties were evaluated at the temperature of the upper surface. A linear regression of $\ln Nu$ on $\ln Ra$ gives

$$\left.\begin{array}{c} Nu = 0{\cdot}396\, Ra^{0{\cdot}227} \\ \text{for} \quad 1{\cdot}89\times 10^3 \leqslant Ra \leqslant 2{\cdot}17\times 10^{12}, \\ 2{\cdot}75 \leqslant Pr \leqslant 6{\cdot}86, \quad 0{\cdot}025 \leqslant L/X \leqslant 0{\cdot}50, \end{array}\right\} \tag{3}$$

where Pr is the Prandtl number ν/α and X is the horizontal length of one side of the fluid layer. This correlation and the experimental data are presented in figure 3. It may be noted that the data in figure 3 display a wavy trend with respect to the correlation (3). This is not believed to be a real feature of the data and is attributed to normal scatter in the single-sample experiments performed in the study. The Prandtl number variation was not included in (3) since most of the data were obtained with $Pr \approx 6{\cdot}5$.

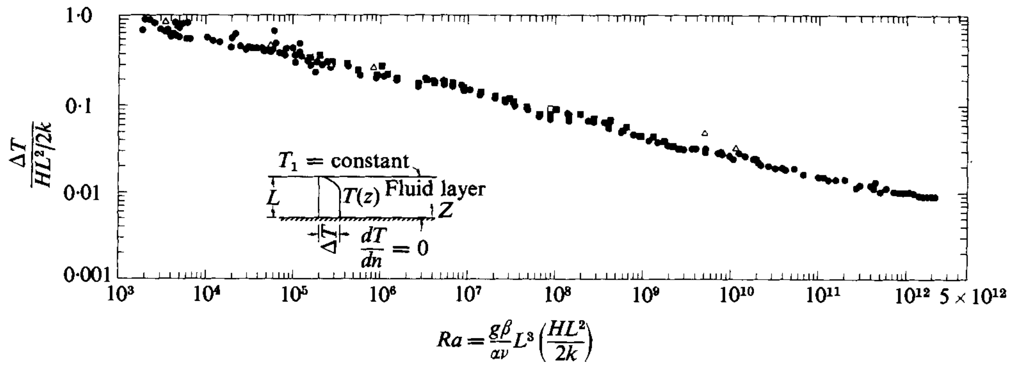


FIGURE 2. Maximum temperature difference across the layer. ●, present study, steady-state experiments; ■, Kulacki & Nagle (1975), steady-state experiments; △, present study, transient experiments; □, Kulacki & Nagle (1975), transient experiments.

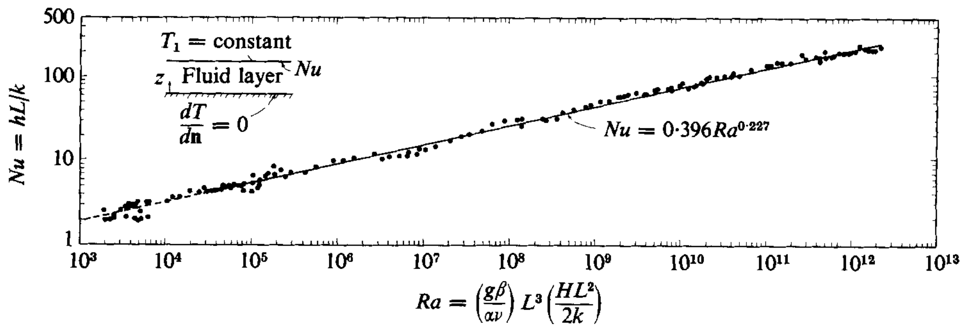


FIGURE 3. Measured Nusselt numbers and heat-transfer correlation, equation (3).

Data for which $Pr < 5$ were for $Ra > 3\times 10^{11}$ and represented 18 out of 133 data points. The experimental data are tabulated by Kulacki & Emara (1975).

It may be noted from figure 3 that the scatter in the measured Nusselt number increases significantly for $Ra < 10^4$. This is due to external effects (e.g. ambient temperature drift over the duration of the experiment) which could not be completely damped out by the small convection apparatus. Convection at $Ra < 10 Ra_c$ is quite feeble, and any externally imposed disturbance or thermal imbalance is difficult to control. In addition, the temperature difference ΔT for low Rayleigh number (laminar) convection is quite small ($\approx 0{\cdot}5$ °C or less in the present experiments), and its measurement is subject to greater inaccuracy than that at higher Rayleigh numbers. It appears, however, that the trend in the Nusselt number is well established for $Ra > 10^4$.

If a correlation of the form of (3) holds in the vicinity of the conduction regime it can be used to measure the critical Rayleigh number by extrapolation to the conduction value of $Nu = 2$. Equation (3) thus predicts a critical Rayleigh number of 1254·3.

To compare existing theoretical values of Ra_c with the value given by (3), it is necessary to take into account the thermal coupling between the fluid layer and its environment. This coupling is expressed by the Biot number $Bi = h_{\text{ext}}L/k$, where h_{ext} is the coefficient of overall heat transfer to the environment. For the sets of apparatus constructed for this study, an equivalent Biot number is defined as the ratio of the thermal conductance of the top plate to that of the fluid layer. Thus

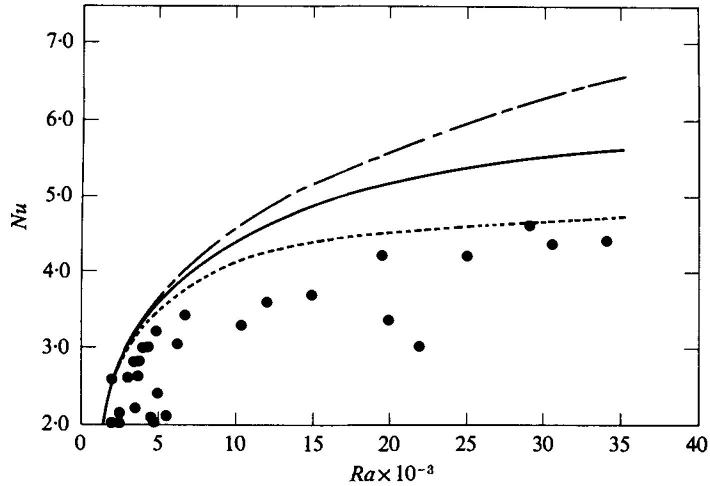


FIGURE 4. Comparison of measured and predicted Nusselt numbers of steady laminar convection. ——, Thirlby (1970), $Pr = 6{\cdot}8$; – – –, Roberts (1967), shape-factor assumption; ------, Roberts (1967), mean-field analysis, $Pr = 6{\cdot}7$; ●, present study, $Pr \approx 6{\cdot}6$.

$Bi = (k_w/L_w)(k/L)^{-1}$, where the conductance k_w/L_w of the upper surface is obtained from the additive thermal conductance concept for a composite slab. For both the large and the small convection chamber, $L = 1{\cdot}27$ cm for the lowest Rayleigh number experiments. Using nominal literature values for the material properties of constant-temperature plates, Bi is 64 for the large chamber and 67 for the small chamber. At a Biot number of 65, the linearized stability theory gives $Ra_c = 1344$ (Kulacki & Goldstein 1975). Thus the value of the critical Rayleigh number given by (3) is within $-6{\cdot}7$ % of the theoretical value. This agreement between the measured and the theoretical critical Rayleigh numbers is quite good since the thermal boundary conditions of the experimental apparatus do not exactly match the idealized thermal boundary conditions of the theory.

The measurements of the critical Rayleigh number of the present study and that of Kulacki & Goldstein (1972) via similar methods in a layer with two constant-temperature boundaries essentially confirm the linearized stability theory as being sufficient for giving limits of instability in volumetrically heated fluid layers.

The heat-transfer results of the present study are in good agreement with the results of Kulacki & Nagle (1975) and in fair agreement with those of Fiedler & Wille (1971), their correlation giving Nusselt numbers about 20 % higher than those of the present work. Combining the data of the present study with those of Kulacki & Nagle gives

$$Nu = 0{\cdot}389\ Ra^{0{\cdot}228} \tag{4}$$

for the same range of Ra, Pr and L/X as in (3). Equation (4) gives a measured value of the critical Rayleigh number of 1314, which is within $-2{\cdot}2$ % of the theoretical value from the linearized stability theory.

In figure 4, the heat-transfer results of the present study are compared with the theoretical results of Roberts (1967) and Thirlby (1970) for convection in rolls. Thirlby presented his results in terms of the quantity M = (mean temperature difference across the layer with no motion)/(mean temperature difference across the layer with motion). Roberts used the reciprocal of this quantity. Since $M = (HL^2/2k)(\Delta T)^{-1}$,

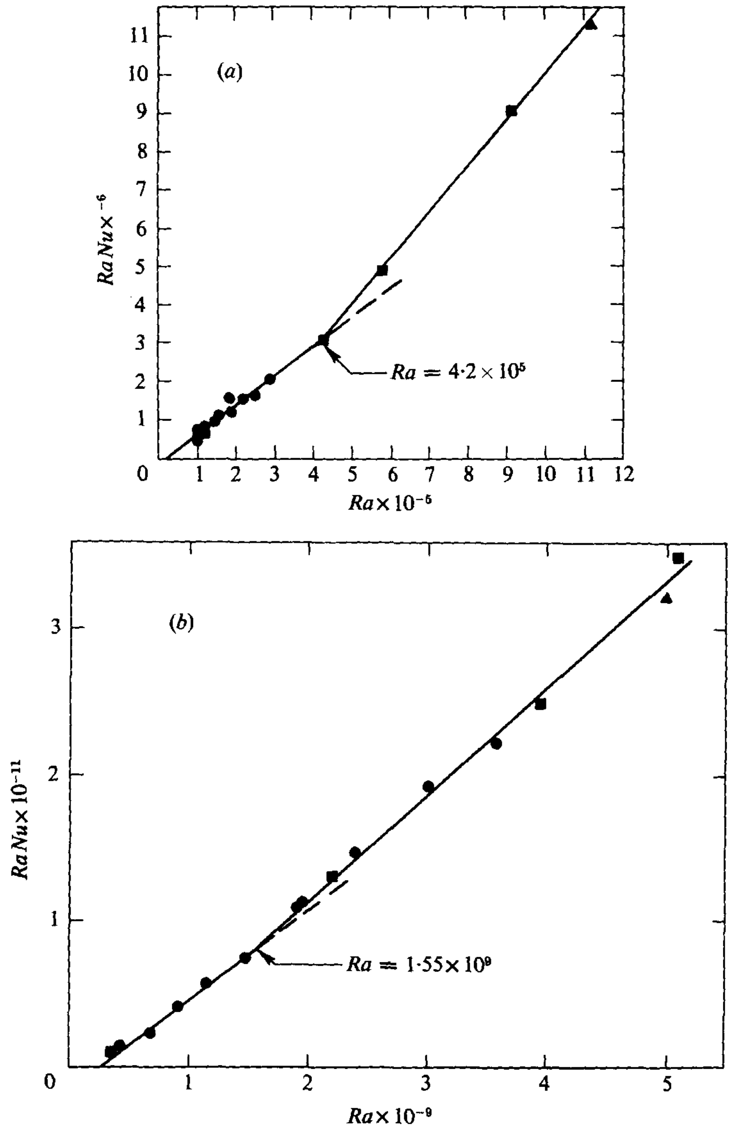
$M = \frac{1}{2}Nu$. The agreement between the experimental Nusselt numbers and Roberts' results obtained using the shape-factor assumption is fair over the entire Rayleigh number range of figure 4, which includes the regimes of feeble and laminar convection. The data and results of both Roberts and Thirlby are in relatively good agreement for $Ra_c \leq Ra \lesssim 4000$. In this range of Ra, the mean temperature profile is still very nearly parabolic, and persistence of a preferred stable planform of motion (e.g. at the critical wavenumber) is possible. For this limited range of Ra, the experiments tend to support the assumptions and hypotheses of both theoretical studies. At higher Rayleigh numbers, agreement between experiment and theory is less good. It is somewhat surprising that Roberts' shape-factor results lie closer to the experimental data than either Thirlby's results, which are the steady-state limit given by a numerical integration of the unsteady conservation equations, or Roberts' mean-field results. More precise experiments and a re-examination of the theory may be needed to resolve these differences. It may be noted that Thirlby's results and the experimental data correspond to slightly different Prandtl numbers, but this difference is not considered significant here.

The heat-transfer data were analysed also for the existence of discrete transitions in heat flux. Such transitions in heat flux have been documented for Bénard convection (Chu & Goldstein 1973) and have been observed to occur also in convection with volumetric energy sources (Kulacki & Goldstein 1972; Kulacki & Nagle 1975). Rayleigh numbers at which transitions in heat flux occur are found by plotting $Nu\,Ra$ *vs.* Ra over a restricted range of Ra in Cartesian co-ordinates. It is the nature of such a plot to smooth the data in a way that accentuates changes in the slopes of straight-line fits to the data. Figures 5 (*a*)–(*d*) presents examples of the transition Rayleigh numbers. The transition Rayleigh numbers of the present study are listed in table 1 along with those of Kulacki & Nagle (1975). The agreement between the transition Rayleigh numbers of the two studies is quite good, and the results of present work essentially confirm those of Kulacki & Nagle. Since both sets of transition Rayleigh numbers were obtained with the same, or a similar, apparatus and for approximately the same Prandtl numbers, this agreement is not surprising. Chu & Goldstein (1973) have discussed this point and the general difficulty of finding agreement between transition Rayleigh numbers for Bénard convection obtained theoretically (when the Prandtl number is eliminated) and experimentally (when different fluids are used).

Transient experiments

A series of runs was performed to measure the development of the temperature profile in the layer when the Rayleigh number was suddenly changed from zero to a value in the laminar regime, then to a value in the transition regime ($Ra \approx 10^5$) and finally to a value in the fully turbulent regime ($Ra \gtrsim 10^9$). A similar series of runs was performed in which the power input to the layer was suddenly turned off after steady convection at a given Rayleigh number had been established.

A graphical summary of the Rayleigh numbers at which transient experiments were conducted is presented in figure 6 with the steady-state Nusselt numbers obtained in these runs. Equation (3) is also plotted in figure 6, and it can be seen that the heat-transfer results of the transient experiments are generally in good agreement with the steady-state results, which were obtained with more closely controlled thermal boundary conditions and much longer flow development times.



FIGURES 5 (a, b). For legend see facing page.

The temperature profiles for developing convection and decaying convection without internal energy sources are presented in figures 7–11. For $Ra = 1{\cdot}18 \times 10^{10}$ and $5{\cdot}01 \times 10^9$, it is seen that in the early stages of flow development the temperature profile develops an excess, or overshoot, in the upper 15–20 % of the layer.† For $Ra = 8{\cdot}55 \times 10^5$ and $2{\cdot}85 \times 10^5$, no local temperature excess is observed. This behaviour for high Rayleigh numbers on heat-up is believed to be caused by the dominant influence of conduction near the upper boundary of the layer in the initial period of flow development. In the remainder of the layer, buoyant forces and mixing effects

† Typically, for experiments at high Rayleigh numbers, the steady-state ΔT was attained approximately 3 h after power had been applied to the layer. The temperature overshoot near the upper boundary was observed in the first 10–20 min.

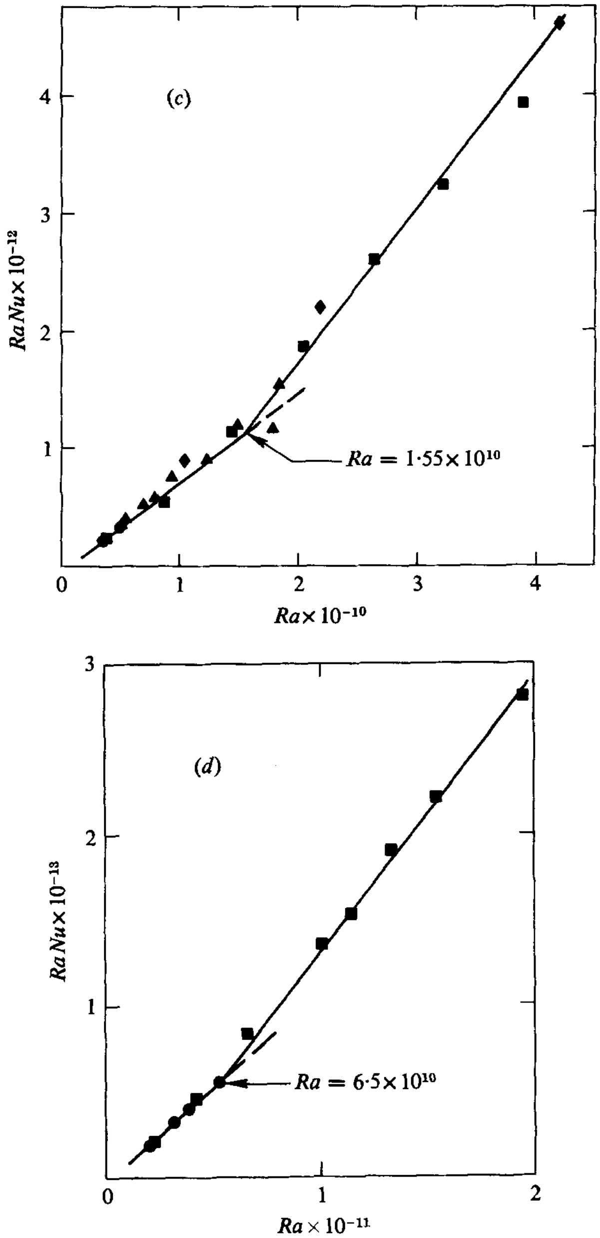


FIGURE 5. Examples of points of discrete transition in heat flux. (*a*) ●, $L = 1{\cdot}905$ cm; ■, $L = 2{\cdot}54$ cm; ▲, $L = 3{\cdot}81$ cm. (*b*) ●, $L = 10{\cdot}16$ cm; ▲, $L = 12{\cdot}70$ cm; ■, $L = 17{\cdot}78$ cm. (*c*) ●, $L = 10{\cdot}16$ cm; ■, $L = 17{\cdot}78$ cm; ▲, $L = 12{\cdot}70$ cm. ◆, $L = 25{\cdot}4$ cm. (*d*) ■, $L = 25{\cdot}4$ cm; ●, $L = 17{\cdot}78$ cm.

are apparently strong enough, even at the start of volumetric heating, to produce the well-mixed isothermal profile characteristic of turbulent convection.

When a step increase in *Ra* is applied such that the final steady state is in the laminar regime of convection ($Ra < 10^5$), no temperature excess is observed near the upper surface. This is most probably due to the less efficient laminar convective

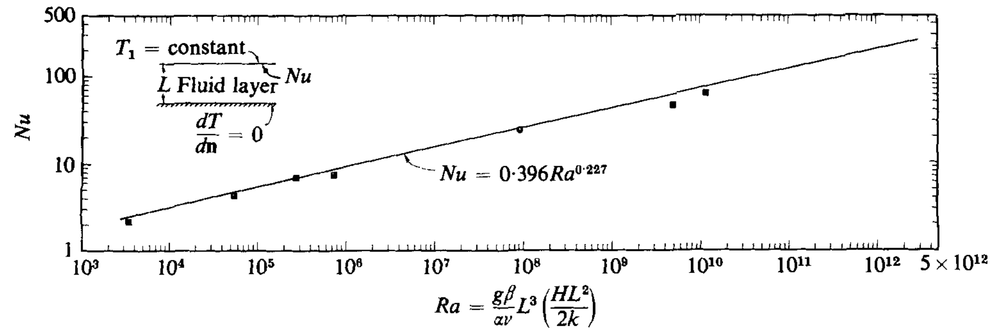


FIGURE 6. Rayleigh numbers of the transient experiments and corresponding measured steady-state Nusselt numbers. The data point (●) at $Ra = 9{\cdot}3 \times 10^7$ is taken from Kulacki & Nagle (1975) and the Nusselt number is computed from their heat-transfer correlation, equation (2).

transport processes in the layer core and the dominance of conductive transport over the entire layer during the early stages of flow development. Furthermore, the core region in this Rayleigh number range does not exhibit a temperature distribution nearly as uniform as that of either transitional or turbulent convection. For very low Rayleigh number laminar convection ($Ra \lesssim 3Ra_c$), the final steady-state temperature distribution maintains the parabolic features of the conduction regime. At a Rayleigh number of $5{\cdot}59 \times 10^4$, the steady-state temperature distribution is, however, relatively uniform over the lower 50 % of the layer.

For decaying convection without internal energy sources, the temperature profiles at all Rayleigh numbers flatten out without exhibiting any local excess or deficit. The initial temperature distribution appears to be of no consequence in the decaying convection process. Local temperature differences in the layer are eliminated by turbulent mixing and molecular diffusion processes which appear to adjust quite rapidly to produce a uniform effect over the layer.

The transient temperature data of this study and of Kulacki & Nagle (1975) permit the development of simple relations between the time required to reach the steady state and the step change in Rayleigh number for both developing and decaying convection. It is found that a relation of the form

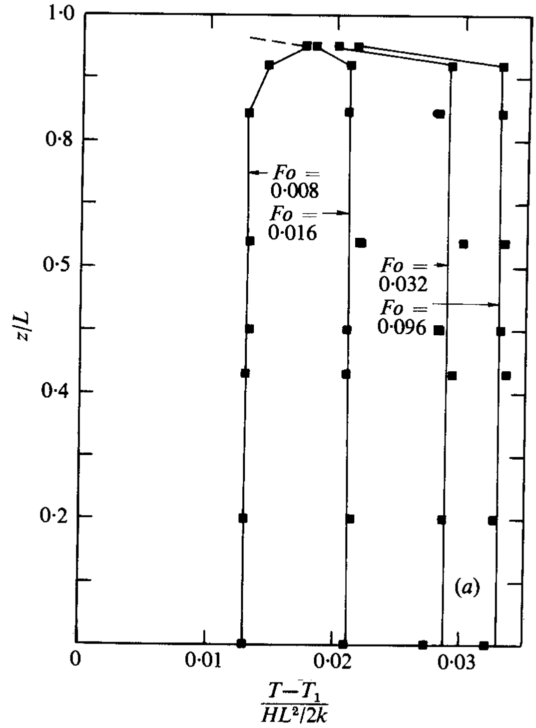
$$Fo = \text{constant} \times (\Delta Ra)^m \tag{5}$$

adequately describes the time required for ΔT to reach its steady-state value for $Ra > 100Ra_c$. In (5), Fo is the Fourier number $\alpha t_{\max}/L^2$, where $t_{\max}$ is the time corresponding to ΔT and ΔRa is the step change (increase or decrease) in the Rayleigh number. The steady-state value of the temperature at any vertical position within the layer is determined from a strip-chart record of the thermocouple response by drawing a tangent to the steady-state limit of the curve. The time at which the probe response becomes within approximately 2 % of the steady-state limit is taken in computing Fo. (See figure 12.)

When a step *increase* in power is applied to the layer, the Fourier number required for the maximum temperature difference to reach its steady-state value is given by

$$Fo_{\max,\,h} = 11{\cdot}577(\Delta Ra)^{-0{\cdot}213}. \tag{6}$$

Equation (6) is presented in figure 13 along with the experimental data.



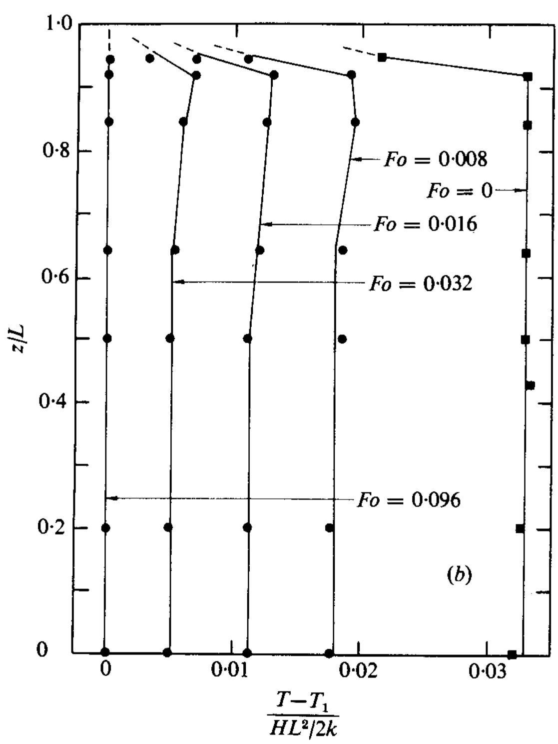
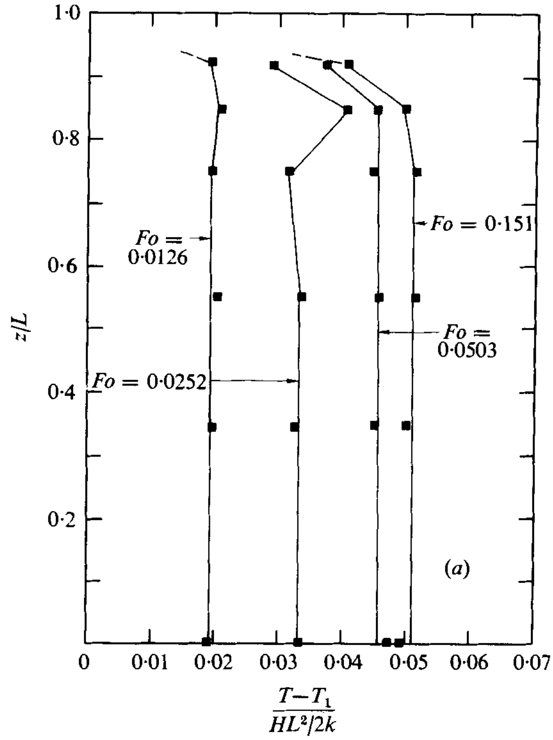


FIGURE 7. Temperature profiles of transient convection due to a step change in Ra. $L = 12{\cdot}7$ cm. (*a*) Initial temperature distribution is $T(z,0) = T_1$; $\Delta Ra = 1{\cdot}18 \times 10^{10}$. (*b*) Initial temperature distribution is that of steady convection at $Ra = 1{\cdot}18 \times 10^{10}$; $\Delta Ra = -1{\cdot}18 \times 10^{10}$.



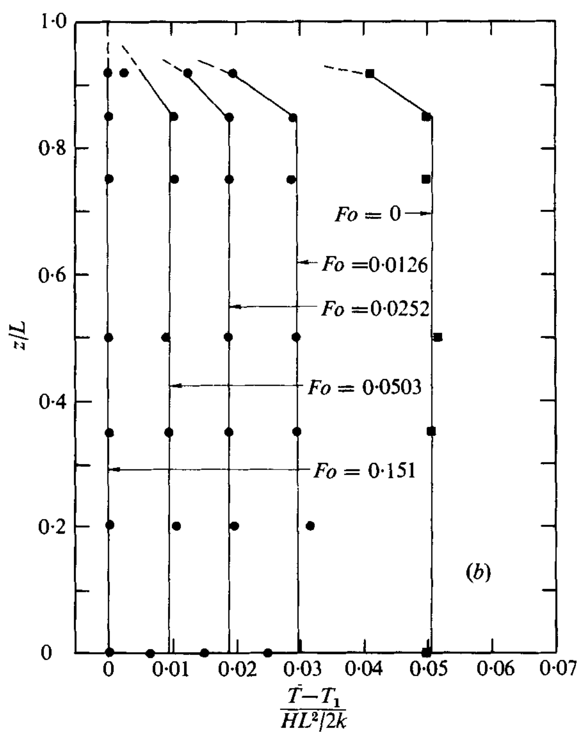
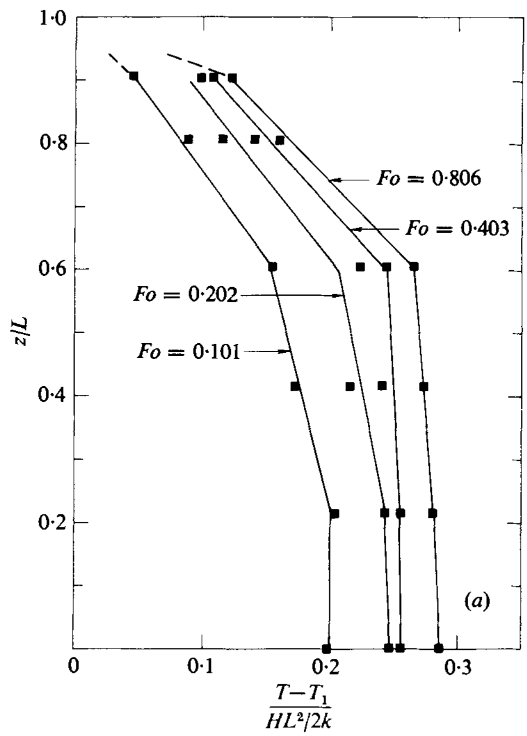


FIGURE 8. Temperature profiles of transient convection due to a step change in Ra. $L = 10{\cdot}16$ cm. (*a*) Initial temperature distribution is $T(z, 0) = T_1$; $\Delta Ra = 5{\cdot}01 \times 10^9$. (*b*) Initial temperature distribution is that of steady convection at $Ra = 5{\cdot}01 \times 10^9$; $\Delta Ra = -5{\cdot}01 \times 10^9$.



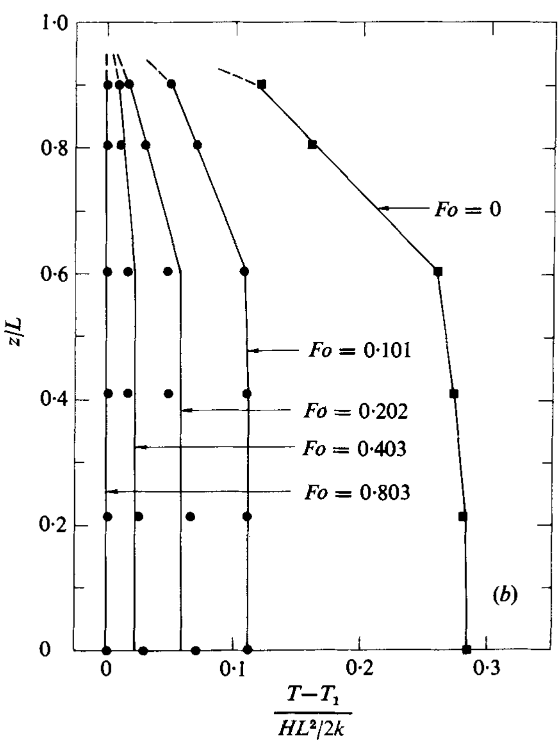


FIGURE 9. Temperature profiles of transient convection due to a step change in Ra. $L = 2{\cdot}54$ cm. (*a*) Initial temperature distribution is $T(z, 0) = T_1$; $\Delta Ra = 8{\cdot}55 \times 10^5$. (*b*) Initial temperature distribution is that of steady convection at $Ra = 8{\cdot}55 \times 10^5$; $\Delta Ra = -8{\cdot}55 \times 10^5$.

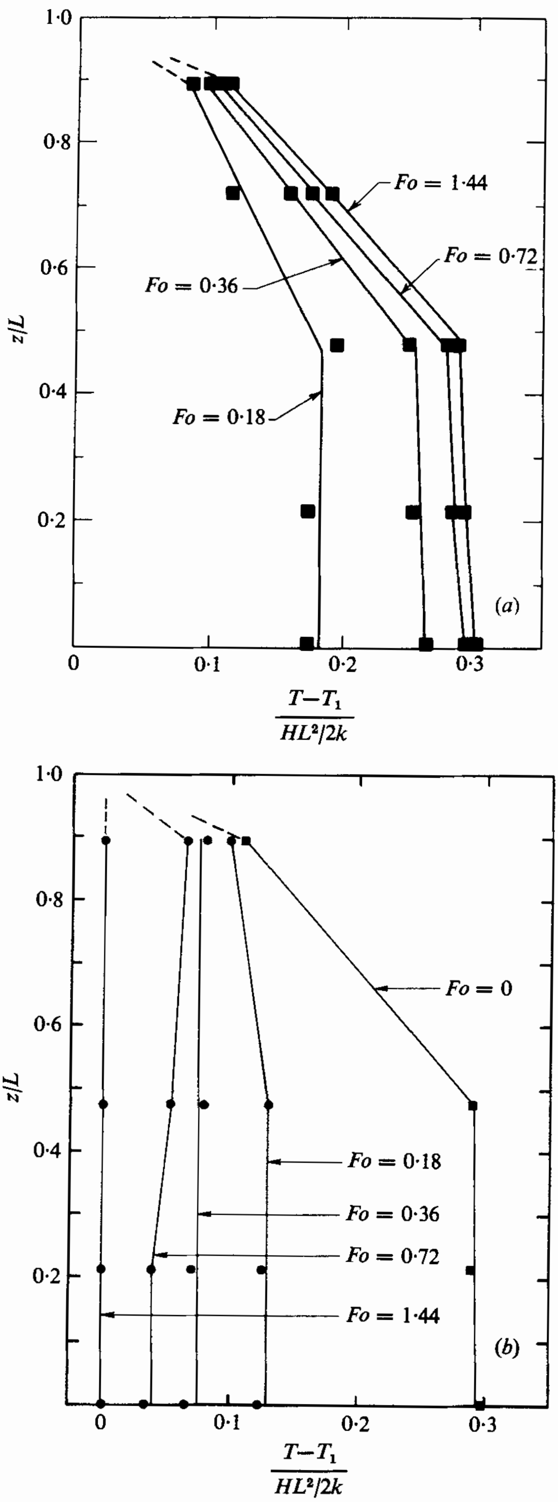


FIGURE 10. Temperature profiles for transient convection due to a step change in Ra. L = 1·905 cm. (a) Initial temperature distribution is $T(z,0) = T_1$; $\Delta Ra = 2{\cdot}85 \times 10^5$. ($b$) Initial temperature distribution is that of steady convection at $Ra = 2{\cdot}85 \times 10^5$; $\Delta Ra = -2{\cdot}85 \times 10^5$.

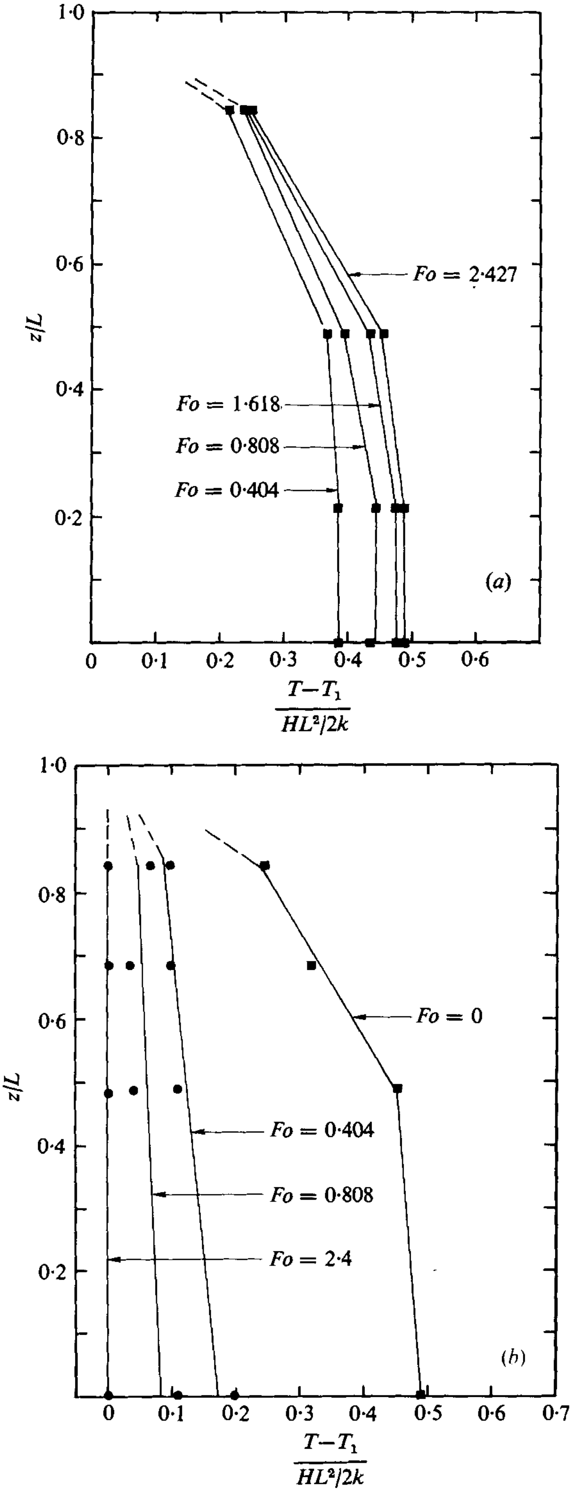


FIGURE 11. Temperature profiles for transient convection due to a step change in Ra. $L = 1{\cdot}27$ cm. (*a*) Initial temperature distribution is $T(z, 0) = T_1$; $\Delta Ra = 5{\cdot}59 \times 10^4$. (*b*) Initial temperature distribution is that of steady convection at $Ra = 5{\cdot}59 \times 10^4$; $\Delta Ra = -5{\cdot}59 \times 10^4$.

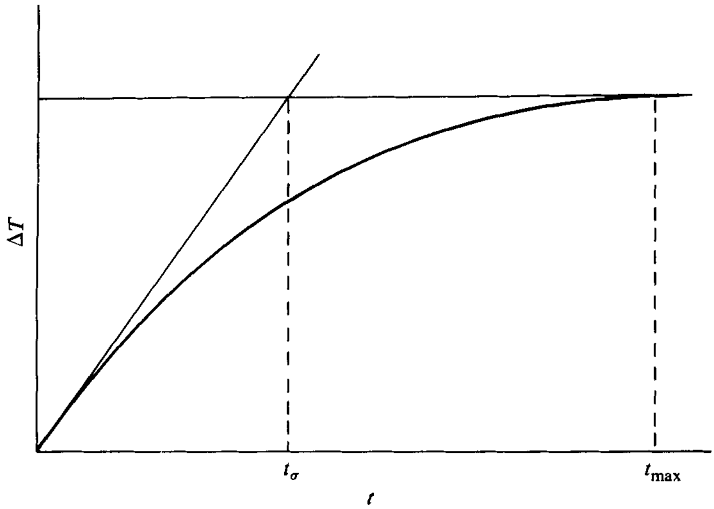


FIGURE 12. Definition of t_{max} and pseudo-first-order time scale t_σ, from thermocouple response.

When a step *decrease* in power is applied to the layer after steady convection has been established at a given Ra, the Fourier number required to reach an isothermal steady state is given by

$$Fo_{max,c} = 11{\cdot}956\,(\Delta Ra)^{-0{\cdot}215}. \quad (7)$$

Another pair of correlations similar to (6) and (7) can be obtained for the pseudo-first-order time scale t_σ, for the developing and decaying convection processes (see figure 12). For a step *increase* in Ra, the Fourier number of the pseudo-first-order time scale for the maximum temperature difference is given by

$$Fo_{\sigma,h} = 2{\cdot}587\,(\Delta Ra)^{-0{\cdot}228}, \quad (8)$$

where $Fo_{\sigma,h}$ is defined in terms of t_σ. For a step *decrease* in Ra, the pseudo-first-order time scale for the maximum temperature difference is given by

$$Fo_{\sigma,c} = 4{\cdot}423\,(\Delta Ra)^{-0{\cdot}275}. \quad (9)$$

It is interesting to note that the time required for the layer to attain the maximum steady-state temperature difference in developing convection with internal energy sources and the time required for the decay of this temperature difference without internal energy sources will be approximately the same [compare (6) and (7)]. Thus it appears that the governing physical parameter of the large-time behaviour of either process is the maximum steady-state temperature difference within the layer. Equations (8) and (9), however, indicate that the initial growth and decay rates of the maximum temperature difference are quite different.

4. Error estimates

The combined uncertainties in the thermophysical properties of the aqueous silver nitrate solutions, geometrical factors (primarily the depth of the layer) and the power consumption per unit volume produced an experimental uncertainty of 5–7 % in the Rayleigh number. The uncertainty in the steady-state Nusselt number was 4–5 %.

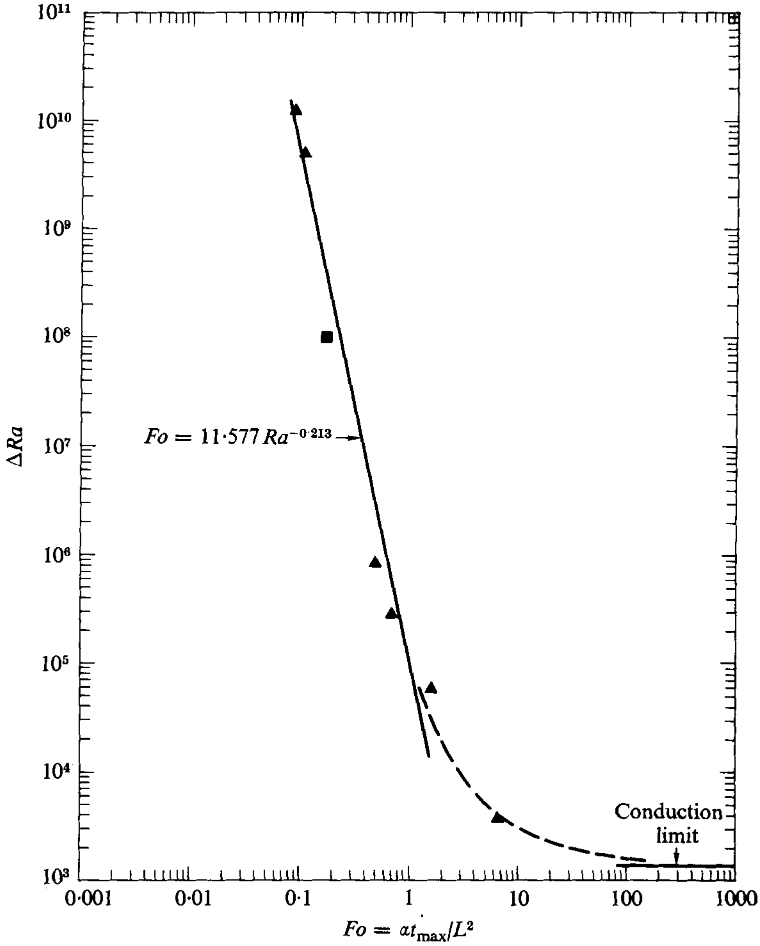


FIGURE 13. Fourier number for development of maximum temperature difference within the layer as a function of the step change in Ra. ▲, present study; ■, Kulacki & Nagle (1975).

Measurements of temperature taken from the strip chart in the transient experiments were considered to be accurate to within 3–5 %. The accuracy of the thermocouple probe was estimated to be of the order of 2–5 %. No detailed calibration was conducted to determine the accuracy of the probe over an extended range of temperatures. Thus the transient temperature measurements were estimated to be accurate to within 5–10 %. While this level of experimental uncertainty is higher than in the steady-state experiments, it is not considered to be unreasonable in view of the technique employed.

The measured values of the layer Fourier number were estimated to have an uncertainty of 2–3 %.

5. Concluding remarks

The steady-state heat-transfer measurements cover the range of Rayleigh numbers $1{\cdot}4 \leqslant Ra/Ra_c \leqslant 1{\cdot}6 \times 10^9$, where $Ra_c = 1344$. This range of Ra includes the laminar, transitional and turbulent regimes of convection. No detailed temperature profile data were obtained in this study which permitted sharp limits on Ra to be obtained for each regime of convection. The steady-state profiles of the transient experiments and the early observations of Tritton & Zarraga (1967) and Schwiderski & Schwab (1971) regarding the breakup of planform flows indicate that the transition from laminar to turbulent convection occurs at $Ra \gtrsim 100Ra_c$. For $Ra \gtrsim 10^7$, mean temperature profiles exhibit the well-mixed, isothermal core characteristic of fully developed turbulent convection.

The heat-transfer data are well represented by a correlation of the form of (3), which is useful for engineering purposes. An alternative correlation is one of the form

$$Nu - 2 = \text{constant} \times (Ra - Ra_c)^m.$$

By combining the data of the present study with those of Kulacki & Nagle (1975) and using (4) to determine the critical Rayleigh number at $Nu = 2$, this correlation is

$$Nu - 2 = 0{\cdot}0787(Ra - 1314)^{0{\cdot}298} \tag{10}$$

for the same range of Ra, Pr and L/X as in (3).

The results of the transient experiments have lead to simple expressions for the time scale for the development of steady convection when internal heating begins suddenly in an initially motionless isothermal layer and the time scale for the complete decay of convection when internal heating is suddenly stopped. Correlations between the layer Fourier number and the step change in Rayleigh number indicate that the governing physical parameter in both processes is the steady-state temperature difference across the layer. Correlations of the form of (5) are found to hold when $|\Delta Ra| \gtrsim 100Ra_c$, i.e. for final and initial steady convection in the turbulent regime. The time scales for the development of the maximum temperature difference and the decay of this temperature difference have been found to be approximately the same for a given ΔRa. No hysteresis effects were observed in the transient experiments, and it is doubtful that the apparatus would have permitted detection of such effects had they been present.

It appears that the local temperature excesses observed in the early period of developing convection occur only for $Ra > 100Ra_c$, i.e. for final steady-state flows which are turbulent. In turbulent convection with uniform volumetric energy sources, eddy transport at the outer edge of the thermal boundary layer on the upper surface is the dominant mode of energy transfer (Kulacki & Goldstein 1972, 1974). The eddy transport process involves the release of cold eddies from the upper surface combined with strong mixing effects near the thermal boundary layer, where the mean temperature profile has a sharp knee. If the release of eddies from the upper surface is viewed as a local instability phenomenon wherein eddy growth and release can be characterized by a local Rayleigh number which exceeds a certain critical value, then the mean temperature distribution near the upper surface must be convectively unstable. When the layer initially experiences volumetric heating, the growth of the temperature difference in the conduction-dominated region near the upper surface proceeds at a rate different from that in the core, where mixing effects produce a lower mean temperature. Thus, with the layer initially convectively stable near the upper surface, eddy transport processes are delayed, permitting the development of a temperature excess. When the mean core temperature becomes large enough, eddy processes begin near the upper surface, and the local temperature excess is eliminated.

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